On Multiple Model Control for Multiple Contact Systems

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Abstract

Multiple contact interaction is common in many robotic applications. These include grasping, wheeled vehicles, and distributed manipulation. All of these applications are capable of experiencing stick/slip phenomena at the contact interfaces. In particular, when there are more kinematic constraints than there are degrees of freedom, some contact interfaces must slip. Moreover, this stick/slip behavior is difficult to predict a priori due to strong sensitivities with respect to friction modeling and normal forces. Motivated by a simple multi-point manipulation device, we discuss how these effects can be accounted for and mitigated using tools from hybrid systems theory and multiple model adaptive control both for analysis and control design. In the context of the multi-point manipulation example, we show how multiple model supervisor-based adaptive control can provide stabilizing controllers for such systems. Our results are validated with simulations that both illustrate the need for these techniques and show their effectiveness.

Key words: Robot Control, Manipulation, Multiple models

1 Introduction

A manipulation system consisting of many points of contact typically exhibits stick/slip phenomenon due to the point contacts moving in kinematically incompatible manners. We call this manner of manipulation *overconstrained manipulation* because not all of the constraints can be satisfied. This paper is concerned with systems that have multiple points of contact, all of which are frictional and adequately described by either constraint forces (when there is

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no slipping at the point contact) or by the slipping reaction force. Prototypes of this situation include multi-point manipulation systems, such as those found in [8,7], as discussed in Section 2.

This paper is organized as follows. Section 2 discusses multi-point manipulation in more detail, and discusses the experimental implementation used before in [8]. Section 3 describes a control approach for this system using multiple model adaptive control [4,1,3,10,2,12] and gives an example simulation for this experimental system when the contact states are assumed to be known perfectly. We then illustrate how variations in the contact state can, not surprisingly, degrade the performance of the algorithm. Section 4 gives the necessary background for understanding stability of switching control system, such as that described in [1,4], and proves the relevant stability properties. We also discuss the implementation of this adaptive control method to the multiple point manipulation example and show in simulation that the original algorithm performance is indeed recovered even when the contact states are not known a priori.

2 Motivation: Multi-point Manipulation

The work in [8] describes an experimental test-bed (seen in Fig.1) that was designed to evaluate and validate control schemes for multiple point manipulation. In such systems friction forces and intermittent contact play an important role in the overall system dynamics, leading to non-smooth dynamical system behavior.

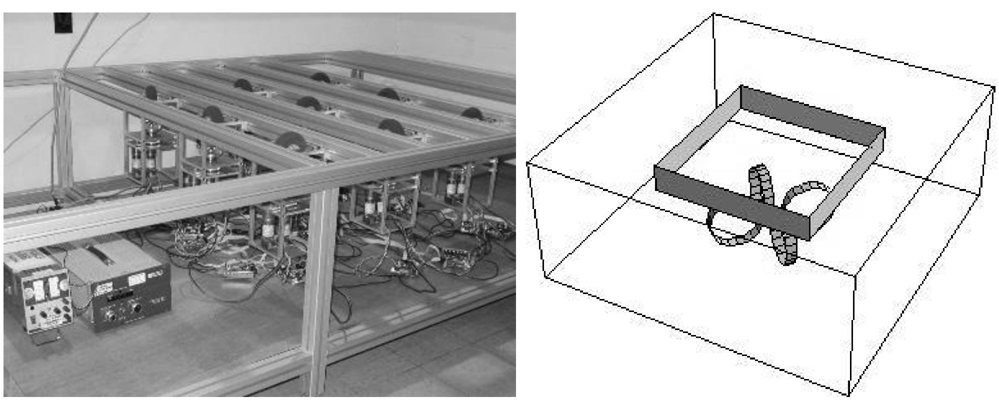

Fig. 1. The Caltech Distributed Manipulation System and *Mathematica* simulation visualization. In the visualization a box is shown supported by four wheels and the bottom and top of the box are transparent to help with visualizing the motion of the wheels. Movies of simulations in this paper can be found at *http://robotics.colorado.edu/∼murphey/MCM*.

A photograph of the apparatus can be seen in Figure 1, along with the visualization we use for simulation purposes. The design is a modular one based on a basic cell design. Each cell contains two actuators. One actuator orients the wheel axis, while the other actuator drives the wheel rotation (see Figure 1(b)). These cells can easily be repositioned within the supporting structure to form different configurations. The system shown in Figure 1(a) is configured with a total of nine cells—though more can be easily added. The position and orientation of the manipulated object is obtained and tracked using a camera. To enable visual tracking, a right triangle is affixed to the moving object. For more details on the experimental setup, please refer to [8].

3 Modeling and Analysis Using Multiple Model Systems

The four-actuator multiple point system seen in Section 2 can be modeled as a hybrid mechanical system with four inputs that can then be reduced to a hybrid kinematic system of the form

$$\dot{\mathbf{x}} = f_1^\sigma u_1^\sigma + f_2^\sigma u_2^\sigma + f_3^\sigma u_3^\sigma + f_4^\sigma u_4^\sigma \tag{1}$$

where $\mathbf{x} = (x, y, \theta)$ is the configuration (of x and y planar translations and of θ planar rotation) and $\sigma \in \{1, 2, 3, 4\}$ is the *contact state* of the system, of which there are in total four. Moreover, $f_3^\sigma = -f_1^\sigma$, $f_4^\sigma = -f_2^\sigma$, and this system can be stabilized to $\mathbf{x} = \{0, 0, 0\}$ by setting $u_1^\sigma = -u_3^\sigma$ and $u_4^\sigma = u_2^\sigma$. Then the system is of the form $\dot{\mathbf{x}} = f_1^\sigma u_1^\sigma + f_2^\sigma u_2^\sigma$, where f_1^σ and f_2^σ are found in Table 1. Also found in Table 1 is a stabilizing control law for each σ found by solving $\dot{V} = -V$ where $V = x^2 + y^2 + \theta^2$. Details of this analysis can be found in [8,9]. Lastly, we assume that σ has a time hysteresis constant due to friction, as is commonly the case [11].

If we define $\eta = ArcTan(\frac{y}{x})$, then the power dissipation method [8,9] predicts that σ has the following dependency on η:

$$\sigma = \begin{cases} 1, & \eta \in (0, \frac{\pi}{2}) \\ 2, & \eta \in (\frac{\pi}{2}, \pi) \\ 3, & \eta \in (\pi, \frac{3\pi}{2}) \\ 4, & \eta \in (\frac{3\pi}{2}, 2\pi) \end{cases} \tag{2}$$

and η is indeterminate on the boundaries. It will be important to our stability results that the system not be able to go to infinity in finite time, which for the system in Eq.(1) is equivalent to requiring that neither u_1 nor u_2 go to infinity. Note that the control law for a given σ only blows up when η is not in the range appropriate for that σ. For instance, $u_1^{\sigma=1}$ blows up when $\eta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, neither of which are conditions that lead to $\sigma = 1$. Moreover, since

σ	Equations of Motion	Control Law ($u_3 = -u_1$ and $u_4 = -u_2$)
1	$\dot{q} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} u_2$	$u_1 = \frac{-k\theta\,(\theta+x-y)+k\left(\theta^2+x^2+y^2\right)}{x+y}$ $u_2 = -k\theta$
2	$\dot{q} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} u_2$	$u_1 = k\theta$ $u_2 = \frac{k\theta\,(\theta+x+y)-k\left(\theta^2+x^2+y^2\right)}{x-y}$
3	$\dot{q} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} u_2$	$u_1 = \frac{k\theta\,(\theta-x+y)-k\left(\theta^2+x^2+y^2\right)}{x+y}$ $u_2 = k\theta$
4	$\dot{q} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} u_2$	$u_1 = -k\theta$ $u_2 = \frac{-k\theta\,(-\theta+x+y)+k\left(\theta^2+x^2+y^2\right)}{x-y}$

Table 1
The four actuator manipulation surface shown in Fig.1 has all kinematic states, many of which are redundant. This figure shows the four distinct equations of motion that can occur in different contact states. This, combined with a choice of hysteresis constant (typically based on friction modeling [11]), completely defines the equations of motion. Note that all the controls grow quadratically with the the state so long as their denominators do not go to zero.

the system is only asymptotically stable, it does not reach the xy origin in finite time. That is, it converges to the xy origin, but $(x(t), y(t)) \neq (0,0)$ for all time. Hence, for small variations of how σ varies with η the controls will remain bounded, and we will assume that this holds for the subsequent results.

Note that the control laws in Table 1 are not only nonlinear, they are not even smooth. In fact, they have discontinuities that coincide with the boundaries and, in particular, discontinuities at the origin (the point towards which we are stabilizing). Also note that σ is observable just from the θ dynamics so long as u_1 $(= -u_3)$ and u_2 $(= -u_4)$ are nonzero and not equal.

If one estimates the model properly, one can get good control performance. For instance, if the coordinate axes are hybrid transitions for the multiple model system (such as happens in the case of uniform friction distributions– see [8]) then the control laws perform quite well. Figure 2 shows a simulation. The initial condition is $\{x_0, y_0, \theta_0\} = \{-1.5, -1, \pi\}$ This, and the other simulations, were all done in *Mathematica*, using Euler integration in order to avoid numerical singularities when crossing contact state bound-

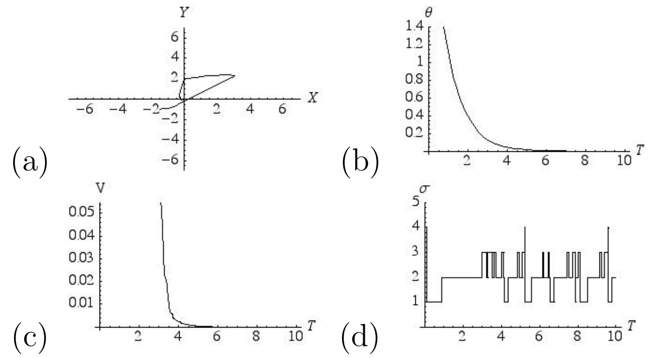

Fig. 2. Simulation of multi-point manipulation when the contact state is known perfectly, a time hysteresis constant due to friction of 0.01s, and σ changes whenever the trajectory crosses the coordinate axes (as predicted by the power dissipation method [8,9]). The bottom four plots are plots of (a) (x, y) trajectory in the plane, (b) θ trajectory versus time, (c) the Lyapunov function versus time, (d) σ, the hybrid state of the system, versus time. Note that the rate of switching for σ does not significantly change over time.

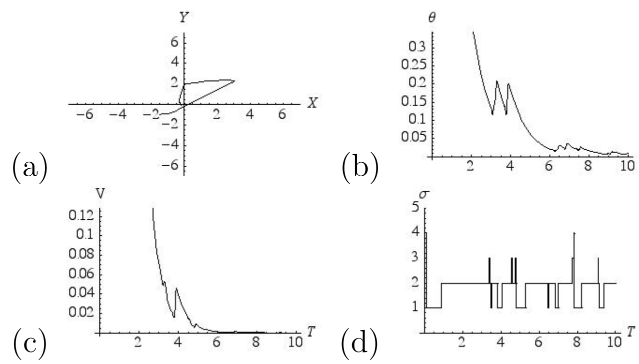

Fig. 3. Simulation of multiple point manipulation when the contact state is estimated is incorrectly estimated. The object is only barely stabilized to the origin due to the contact state being varying from the nominal value and the Lyapunov function does not monotonically decrease. However, the rate of switching in σ does not change significantly.

aries. (A movie of this simulation and other simulations may be found at *http://robotics.colorado.edu/∼murphey/MCM.*) In Fig. 2, the object is stabilized to $(x, y, \theta) = (0, 0, 0)$ with no difficulty. It is worth noting that the addition of noise could potentially make this system hit the origin. However, because all the configurations near the origin result in finite input magnitudes, we simply use the fact that the probability of hitting the origin due to noise is zero. Because of this, we do not consider the effect of additive noise in our model. If, however, the configurations where the controller goes to infinity had finite measure, more consideration would need to be made. Note also that the Lyapunov function is monotonically decreasing, and the switching signal σ continues switching, but does not increase the rate at which it switches. In the simulation in Fig.2 its average rate of switching is typically less than 20 switches per second, but by our assumptions on a friction hysteresis of $0.01s$

the switching cannot be more than 100 switches per second. Hence, our stability results will often be conservative, as they rely on this upper bound, which the system does not typically approach.

In the simulations the constraints are enforced separately from the control law, allowing the control to switch at different times from the constraints. In particular, if the boundary that determines the physical contact state is allowed to vary while the control laws only change at the estimated boundaries, then the performance degrades substantially. Starting the object at an initial condition of $\{x_0, y_0, \theta_0\} = \{-1.5, -1, \pi\}$, Fig. 3 shows this degradation in comparison to Fig. 2, although the system is still stable. In the case of Fig. 3, the controller is assuming that the contact state changes when the center of mass of the object crosses the line $x = 0$, whereas the contact state is actually changing when the line $x = -0.3y$ is crossed. This is precisely the difficulty fixed in Section 4.

4 Stability Conditions

Now we move on to set this problem up more formally, roughly following [5,1,4]. Suppose that we have a family of plants indexed by $p \in \mathbb{P}$, all of which are stabilized by a control law with Lyapunov function V_p. Switching between plants is governed by the switching signal σ. Such systems can be written as:

$$\dot{x} = F_{\sigma(\mathbf{x},t)}(\mathbf{x}, t) \qquad \sigma(\mathbf{x}, t) \in \mathbb{P} \tag{3}$$

where $\mathbb{P}$ is an index over the set of all admissible plants. This system is therefore a generalization of Eq. (1). We assume that the F_p satisfy the following Lyapunov criteria; that there exist for all $p \in \mathbb{P}$ differentiable functions $V_p : \mathbb{R}^n \to \mathbb{R}$, positive constants λ_0, γ and class $\mathcal{K}_\infty$ functions $\alpha, \overline{\alpha}$ satisfying:

$$\dot{V}_p = \frac{\partial V_p}{\partial \mathbf{x}} F_q \leq -2\lambda_0 V_p \ for\ p = q, \tag{4}$$

$$\dot{V}_p = \frac{\partial V_p}{\partial \mathbf{x}} F_q \leq 2\lambda_{F'} V_p \ for\ p \neq q, \tag{5}$$

$$\alpha(\|\mathbf{x}\|) \leq V_p(\mathbf{x}) \leq \overline{\alpha}(\|\mathbf{x}\|), \tag{6}$$

$$V_p \leq \gamma V_q, \tag{7}$$

for all $\mathbf{x} \in \mathbb{R}^n$ and $p, q \in \mathbb{P}$. These are relatively standard requirements for Lyapunov functions, except for the condition in Eq.(5) (which requires that whenever the plant and the controller are not matched the resulting instability is bounded by some growth rate). Also, note that for the example system in Section 2 these conditions hold trivially with $\alpha = \overline{\alpha} = Id$, $\gamma = 1$, and $\lambda_{F'} \leq \sqrt{u_1^2 + u_2^2}$ (which is bounded if σ's dependence on η does not cause the controls to blow up). However, for general manipulation problems this will not typically hold, particularly when a common Lyapunov function does not exist, thereby causing $\gamma < 1$.

Switching signals σ are assumed to be a piecewise continuous (and therefore measurable) function coming from a family of functions S. We say that Eq.(3) is *uniformly exponentially stable over* S if there exist positive constants c and λ such that for any $\sigma \in S$ we have

$$\|\Phi_\sigma(t,\tau)\| \le ce^{-\lambda(t-\tau)} \qquad \forall t \ge \tau \ge 0.$$

Here $\Phi_\sigma(t,\tau)$ denotes the flow (given σ) of Eq.(3). For such a system we say that λ is its *stability margin*.

To characterize and distinguish different families of functions S, we employ the following definitions (from [5]). Given $\sigma \in S$, we define $N_\sigma(t,\tau)$ to be the (integer) number of switches or discontinuities in σ in the interval (t,τ). Given two numbers τ_{AD} and N_0, called the *average dwell time* and *chatter bound* respectively, we say that $S_{\text{ave}}[\tau_{AD}, N_0]$ is the set of all switching signals satisfying $N_\sigma(t,\tau) \le N_0 + \frac{t-\tau}{\tau_{AD}}$. We will assume for the rest of the present work that switching signals σ_e (the external switching determining the contact state) can be characterized in this way.

Assumption 4.1 *Assume σ_e switching satisfies*

$$N_{\sigma_e}(t,\tau) \le N_0^e + \frac{t-\tau}{\tau_{AD}^e}$$

for some $N_0^e > 0$ and τ_{AD}^e.

This assumption can be physically related to the example system in Section 2 through two main facts. First, switching only occurs when the system crosses the coordinate axes. Hence, if the system is spiraling in towards the origin the average dwell time will be reasonably long. Secondly, the frictional hysteresis bounds the average dwell time. Note that, if necessary, we can similarly require that the signal σ_c (the switching signal that dictates the current controller) also satisfy dwell-time requirements (i.e., $N_{\sigma_c}(t,\tau) \le N_0^c + \frac{t-\tau}{\tau_{AD}^c}$) to ensure that the control switching does not destabilize the system.

4.1 Nominal Stability Conditions

It is well known that switching between a set of stable linear systems may well yield an unstable system [6]. This means that even in the most moderate case, where estimation of the contact state is perfect and there are no latencies in sensing or actuation, our multiple contact system can in principle be destabilized by switching contact state. Our purpose in this section is to apply some results from the theory of switching systems to understand physically meaningful conditions that will guarantee stability for a multiple model system (even those without a common Lyapunov function). In particular, we will characterize such a condition in terms of the average dwell time as it was described above.

First, the following result from [5] will be helpful. It states that for a collection of stable plants as Eq.(3) a bound on the average dwell time can be determined such that the hybrid system is stable.

Lemma 4.1 ([5]) *Given a system of the form in Eq. (3) such that Eqs. (4), (6), and (7) hold, there is a finite constant* τ^*_{AD} *such that Eq.(3) is uniformly exponentially stable over* $S_{ave}[\tau_D, N_0]$ *with stability margin* $\lambda \in (0, \lambda_0)$ *for any average dwell time* $\tau_{AD} \geq \tau^*_{AD}$ *and any chatter bound* $0 < N_0$*.*

In particular, the average dwell time must satisfy $\tau_{AD} > \frac{\log \gamma}{2(\lambda - \lambda_0)}$. Note that if we have a common Lyapunov function, then $\gamma = 1 \Rightarrow \log \gamma = 0 \Rightarrow \tau_{AD} = 0$ satisfies the stability requirements. Hence, common Lyapunov functions are highly desirable, if they can be found. A corollary of this result relevant to the multiple point contact example is Corollary 4.2.

Corollary 4.2 *If the system in Eq. (1) satisfies:*

(1) It is stabilized with a quadratic Lyapunov function V_p *for every* p*;*
(2) The switching signals are equal $\sigma_e = \sigma_c$ *(i.e., the observer is perfect);*
(3) The physical hysteresis is such that $\sigma_e \in S_{ave}[\frac{\log \gamma}{2(\lambda - \lambda_0)}, N_0]$ *for some* N_0*;*
(4) The environmental switching signal σ_e *varies according to the rule in Eq.(2);*

then Eq. (1) is exponentially stable with stability margin λ*. Moreover, for the particular choice of controllers shown in Table 1, the system is stabilized for any* $\tau^e_{AD} > 0$*.*

What does this mean for a multiple model system where there are external signals determining the switching, such as is the case in a multiple contact system? It means that so long as there are no latencies, no errors in estimation, and no noise in the sensors, the multiple model system is stable so long as the external switching signals σ_e are kept sufficiently slow on the average. How slow depends on how the controllers for each plant are designed and, more importantly, how they are related to each other. The closer γ can be kept to 1, the more quickly σ_e may switch without destabilizing the system.

What happens if there is noise, latencies, and time delays causing the controller switching σ_c to not coincide with the environmental switching σ_e? Most of these issues are adequately addressed in [1,4]. However, if $\sigma_c \neq \sigma_e$, fundamental instabilities due to temporary mismatch between controllers and plants can occur. As mentioned in [4], even traditional adaptive control systems with smoothly varying parameters may have situations where the supervisor switches to a controller whose feedback connection with the current plant is unstable. In our case this can happen because the physics change before the controller can change due to time delays. The next section is dedicated to understanding the consequences of controller/plant mismatch. In it, we extend Lemma 4.1 to the case where there is a time delay during which the controller/plant feedback connection is unstable. The basic result of the analysis is roughly that the longer the time delay, the slower the external switching must be in order to maintain stability.

4.2 Stability with Controller/Plant Mismatch

We now move on to address how to guarantee stability in the case where our system switches between two types of plants, one stable and one potentially unstable. The stable plants correspond to the case where the controller is properly chosen to be stable at a particular time. The potentially unstable plants correspond to the case where, due to any number of factors such as time-delays, actuation latencies, controller design, etcetera, the feedback connection is unstable.

Assume we have equations of motion of the following form:

$$\dot{\mathbf{x}} = \begin{cases} F'_q(\mathbf{x}, t) \text{ on } [t_i, t_i + d_\sigma) \\ F_p(\mathbf{x}, t) \text{ on } [t_i + d_\sigma, t_{i+1}) \end{cases} \tag{8}$$

where (for each p) $\dot{\mathbf{x}} = F_p(\mathbf{x})$ is asymptotically stable and (for each q) $\dot{\mathbf{x}} = F'_q(\mathbf{x})$ is potentially unstable but has a bound on the rate of growth $\lambda_{F'}$. Note that our example system in Section 2 satisfies these requirements because all the F_p are stable by design. Again, let $\{t_0, \ldots, t_n\}$ denote the times we switch away from a plant of the form F_p and assume that for some time d_σ a plant of the form F'_q describes the state evolution. Then, at time $t_i + d_\sigma$ the system switches back to a plant of the form F_p. This is exactly what will happen if the physical switching sequence σ_e does not exactly coincide with the control switching sequence σ_c.

It is now useful to extend Lemma 4.1 to accommodate d_σ. The resulting tradeoff is not surprising–the larger d_σ becomes, the more slowly σ_e is allowed to switch. In particular, we find that in the course of the following proof that choosing

$$\tau^e_{AD} > \frac{\frac{\log \gamma}{2} + 2\lambda_{F'} d_\sigma}{(\lambda_0 - \lambda)} \tag{9}$$

where τ^e_{AD} is the average dwell time of the switching signal σ_e, γ is the bound on the Lyapunov functions for the stable plants $F_p \;\; p \in \mathbb{P}$, $\lambda_{F'}$ is the maximum rate of growth across all $F'_q \;\; q \in \mathcal{Q}$, λ_0 is the minimum stability margin for $F_p \;\; p \in \mathbb{P}$, and λ is a number chosen in the interval $[0, \lambda_0)$.

Lemma 4.3 *Given a system of the form in (8) such that all the F_p satisfy Eqs. (4), (5), (6), and (7), there is a finite constant τ^*_{AD} and a finite constant d_σ such that Eq.(8) is uniformly exponentially stable over $\mathcal{S}_{ave}[\tau_{AD}, N_0]$ with stability margin λ, for any average dwell time $\tau_{AD} \geq \tau^*_{AD}$, any chatter bound $0 < N_0$.*

PROOF. We follow the basic methodology laid out in [5], adapting it for the case of delays causing incompatibility between the current plant determined by σ_e and the current controller determined by σ_c. Assume an arbitrary switching signal σ_e that on an interval $[t_0, T]$ is discontinuous at times:

$$\{t_0, t_1, t_2, \cdots, t_{N_{\sigma_e}(t_0,T)-1}, T\}.$$

Now we define

$$v(t) = e^{2\lambda_0 t} V_{\sigma(t)}(\mathbf{x}(t)).$$

This function is piecewise differentiable, so

$$\begin{aligned}
\dot{v} &= 2\lambda_0 v + e^{2\lambda_0 t} \frac{\partial V_{p_i}}{\partial \mathbf{x}} F'_{p_i} \;\; for \; t \in (t_i, t_i + d_\sigma) \\
\dot{v} &= 2\lambda_0 v + e^{2\lambda_0 t} \frac{\partial V_{p_i}}{\partial \mathbf{x}} F_{p_i} \;\; for \; t \in (t_i + d_\sigma, t_{i+1})
\end{aligned}$$

$\forall i < N_{\sigma_e}(t_0, T)$. This implies that

$$\begin{aligned}
v(t) &\leq e^{2\lambda_{F'} d_\sigma} v(t_i) \;\; for \; t \in (t_i, t_i + d_\sigma) \\
v(t) &\leq v(t_i + d_\sigma) \;\; for \; t \in (t_i + d_\sigma, t_{i+1})
\end{aligned}$$

$\forall i < N_{\sigma_e}(t_0, T)$. Therefore, we have that

$$v(t) \leq e^{2\lambda_{F'} d_\sigma} v(t_i) \;\; for \; t \in (t_i, t_{i+1})$$

$\forall i < N_{\sigma_e}(t_0, T)$. Moreover,

$$\begin{aligned}
v(t) &= e^{2\lambda_0 t_{i+1}} V_{p_{i+1}}(\mathbf{x}(t_{i+1})) \\
&\leq \gamma e^{2\lambda_0 t_{i+1}} e^{2\lambda_{F'} t_{i+1}} V_{p_i}(\mathbf{x}(t_{i+1})) \\
&= \gamma e^{2\lambda_0 t_{i+1} + 2\lambda_{F'} t_{i+1}} V_{p_i}(\mathbf{x}(t_{i+1}))
\end{aligned}$$

Due to the fact that V_{p_i} is continuous, we get that

$$v(t_{i+1}) \leq \gamma e^{2\lambda_{F'} t_{i+1}} v(t_{i+1}).$$

Iterating from $i = 1$ to $i = N_{\sigma_e}(t_0, T)$, we get

$$v(T) \leq \gamma^{N_{\sigma_e}(t_0,T)} e^{2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T)} v(t_0).$$

This gives us

$$e^{2\lambda_0 T} V_{\sigma(T)}(\mathbf{x}(T)) \leq \gamma^{N_{\sigma_e}(t_0,T)} e^{2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T)} V_{\sigma(0)}(\mathbf{x}(0))$$

$\Rightarrow$

$$V_{\sigma(T)}(\mathbf{x}(T)) \leq e^{-2\lambda_0(T-t_0)+N_{\sigma_e}(t_0,T)\log\gamma+2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T)} V_{\sigma(0)}(\mathbf{x}(0)).$$

Therefore,

$$\|\mathbf{x}(T)\| \leq \overline{\alpha}(e^{-2\lambda_0(T-t_0)+N_{\sigma_e}(t_0,T)\log\gamma+2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T)} \alpha(\|\mathbf{x}(t_0)\|)).$$

which, for a constant q (determined in [5], implies that

$$\|\mathbf{x}(T)\| \leq q e^{-2\lambda_0(T-t_0)+N_{\sigma_e}(t_0,T)\log\gamma+2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T)} \|\mathbf{x}(t_0)\|.$$

To get a stability margin of λ one need only satisfy the following equation:

$$-2\lambda_0(T-t_0) + N_{\sigma_e}(t_0,T)\log\gamma + 2\lambda_{F'} d_\sigma N_{\sigma_e}(t_0,T) \leq k - \lambda(T-t_0)$$

for some value of $k > 0$. This is equivalent to

$$N_{\sigma_e}(t_0,T)(\frac{\log\gamma}{2} + 2\lambda_{F'} d_\sigma) \leq k - \lambda(T-t_0),$$

which is equivalent to

$$N_{\sigma_e}(t_0,T) \leq \frac{k + (\lambda_0 - \lambda)(T-t_0)}{\frac{\log\gamma}{2} + 2\lambda_{F'} d_\sigma}.$$

If we set

$$N_0 = \frac{k}{\frac{\log\gamma}{2} + 2\lambda_{F'} d_\sigma} \qquad \tau_{AD} = \frac{\frac{\log\gamma}{2} + 2\lambda_{F'} d_\sigma}{\lambda_0 - \lambda}$$

we get the condition that

$$N_\sigma(t_0,\tau) \leq N_0 + \frac{T-t_0}{\tau_{AD}}, \tag{10}$$

so whenever the triple $(N_0, \tau_{AD}, d_\sigma)$ satisfies the relation in Eq. (10), we have a stable system with stability margin λ. Just as in [5], by choosing k sufficiently large, we can accommodate any chatter bound. $\square$

The following corollary is an immediate consequence of Lemma 4.3.

Corollary 4.4 *If the system in Eq. (1) satisfies:*

(1) It is stabilized with a quadratic Lyapunov function V_p for every p;
(2) The time delay between σ_e and σ_c is less than some d_σ and τ_{AD} and d_σ satisfy Eq.(9);
(3) The physical hysteresis is such that $\sigma_e \in S_{ave}[\frac{\frac{\log\gamma}{2}+2\lambda_{F'}d_\sigma}{(\lambda_0-\lambda)}, N_0]$ for some N_0;
(4) $u_1(t)$ and $u_2(t)$ are finite for all time;

then Eq. (1) is exponentially stable with stability margin λ. Moreover, for the particular choice of controllers shown in Table 1, the system is exponentially stabilized.

Corollary 4.4 indicates that if the contact states change slowly enough (i.e., τ_{AD}^e is large) and supervisory feedback is fast enough (i.e., d_σ is small), then the system can be controlled using an estimate from an estimator $\mathbb{E}$ that is estimating the contact state on-line. Among other things, this means that one does not have to concern oneself with the friction model to establish where switching occurs. Instead, the contact states can change arbitrarily, so long as they do so sufficiently slowly on the average. Also note that if there is a common Lyapunov function and $d_\sigma = 0$, then $\log\gamma = \log 1 = 0$, and the system will be stable for any switching signal. In situations where this is not the case, it would be useful to know if a combination of physical geometry and controller choice can guarantee a lower bound on τ_{AD}^e, but for now we leave it as a standing assumption that it can be bounded (typically based on our understanding of the hysteresis due to friction).

4.3 Simulation and Verification using Multiple Model Control

Now we apply the results of the previous analysis to the example in Fig.1, 2, and 3.

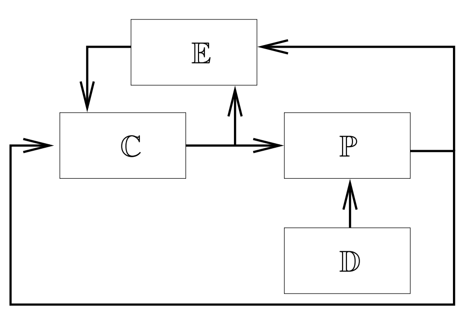

Fig. 4. A multiple model control system

Efforts in the adaptive control community have already created a framework appropriate to addressing the problem of estimating and accommodating changes in contact state. In particular, *supervisory control* (as in [4,1,3,10,2,12] and elsewhere) is an effective technique to use when a system is a linear multiple model system. Fortunately, our system, when reduced to a kinematic system using the power dissipation method, is a first order system with constant vector fields. (In fact, not only is it linear, it does not even have drift.) When one includes the feedback laws, the system is nonlinear and nonsmooth, but satisfies all the Lyapunov assumptions found in Eqs. (4), (5), (6), and (7). Hence, this supervisory framework easily answers how to estimate the current contact state based on the output (in this case the entire state $\mathbf{x} = \{x, y, \theta\}$) of the system as well as stabilize the outputs to $\mathbf{x} = \{0, 0, 0\}$.

Consider the block diagram representation of a supervisory control system found in Fig. 4. Denote the set of possible admissible plants by $\mathbb{P}$. Each model in $\mathbb{P}$ represents a contact state of the overconstrained system. Assume that associated with each plant P_σ coming from $\mathbb{P}$ there is a known stabilizing controller C_σ. Denote the set of these controllers by $\mathbb{C}$. To determine which model in $\mathbb{P}$ most closely "matches" the *actual* model, the input-output relationships for all the plants in $\mathbb{P}$ will need to be estimated. Hence, the need for the *estimator*, denoted by $\mathbb{E}$, which will generate errors between the predicted output for each plant and the actual output of the multiple model system. $\mathbb{E}$ literally compares the estimated output of every model to the actual output (in this case (x, y, θ)) and chooses which model most closely matches the output (using the metric $\|\mathbf{x}\|^2 = x^2 + y^2 + \theta^2$). For a multiple model system with a common Lyapunov function, $\mathbb{E}$ typically looks like

$$\mathbb{E}(y) = \arg\min_{\sigma} \|y_\sigma - y\|$$

where y_σ is the computed output of a plant $p_\sigma \in \mathbb{P}$ and y is the physical output. (Hence, the output of $\mathbb{E}$ is the σ that best represents the systems dynamics at any given time.) More sophisticated version of this estimator exist for systems that do not have common Lyapunov functions [4], but they are basically simple adaptations of this equation. Based on $\mathbb{E}$, the controller chooses the controller. Additionally, there is an environmental signal generator $\mathbb{D}$ creating σ_e. $\mathbb{D}$ represents the externally driven switches in contact state.

Now apply this supervisory approach to the four actuator array from Section 3. Replace the boundary $x = 0$ with the boundary $x = -0.3y$, and allow the estimator $\mathbb{E}$ to estimate the contact state. In this case (found in Fig. 5) the performance is considerably better than that found in Fig. 3 and resembles the performance found in Fig. 2. In these simulations we have found that adding relatively small amounts of sensor noise to the output and time delay in the system does not substantially affect the performance, which corroborates the results in [1,4] on the robustness of this approach.

We must emphasize that although the previous analysis assumed the existence of a stabilizing controller for each plant (hence allowing us to treat p

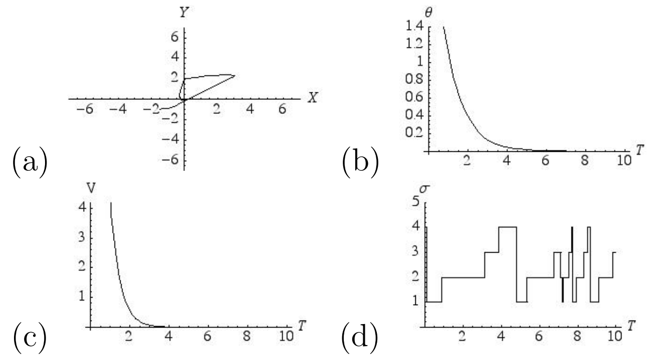

Fig. 5. Simulation of multiple point manipulation when the contact state is estimated on-line, using the supervisory control methodology. A time-delay of $d_\sigma = 0.05$ is used in this simulation. Here the performance is much closer to that seen in Fig. 2, the case where our knowledge of the state is perfect, including the fact that the Lyapunov function now monotonically decreases. Moreover, the rate of switching for σ does not change significantly.

stable plants), we cannot naively design the estimator for determining which control law to use. When two models in $\mathbb{P}$ are very closely related, their outputs will be very close. Hence, small amounts of sensor noise can cause rapid switching between the "best" fit for the output signal. This rapid switching could then destabilize the system, particularly if the controllers for each plant were designed independently of each other. This can be addressed using a generalization of Fig. 4 found in [4,1].

5 Conclusions

In this paper we have introduced the use of multiple model adaptive control [1,4,6] for stabilization of manipulation problems that involve multiple contacts. We show in simulation that this technique works well in the context of a simple example (based on prior experimental work [8]). The problem of contact state estimation and accommodation is clearly important for systems in which stick/slip phenomena play a dominant role. Indeed, for the multiple point manipulation experiment described here, manipulation tasks are actually impossible without the constant trade-off between sticking and slipping. Moreover, the model/controller presented here does not include any explicit model of friction, making the proposed techniques applicable to cases where an unstructured environment makes it unlikely that one can model frictional interactions accurately.

Ultimately, the analytical techniques presented here should be extended to the more geometric setting of grasping and manipulation in the presence of gravitational forces. In particular, cases where a common Lyapunov does not exist should be examined in depth using the analytical techniques developed here.

In the meantime, these results will be implemented on a second generation version of the experiment discussed in Section 2.

References

[1] B.D.O. Anderson, T.S. Brinsmead, F. De Bruyne, J.P. Hespanha, D. Liberzon, and A.S. Morse. Multiple model adaptive control. I. finite controller coverings. *George Zames Special Issue of the Int. J. of Robust and Nonlinear Control*, 10(11-12):909–929, Sep 2000.

[2] M. Athans, D. Castanon, K.P. Dunn C.S. Greene, W H W.H. Lee, N.R. Sandell, and A.S. Willsky. The stochastic control of the F-8C aircraft using a multiple model adaptive control method. *IEEE Transactions on Automatic Control*, 22(5):768–780, October 1977.

[3] M.K. Ciliz and K.S. Narendra. Adaptive control of robotic manipulators using multiple models and switching. *International Journal of Robotics Research*, 15(6):592–610, 1996.

[4] J.P. Hespanha, D. Liberzon, A.S. Morse, B.D.O. Anderson, T.S. Brinsmead, and Franky De Bruyne. Multiple model adaptive control, part 2: Switching. *Int. J. of Robust and Nonlinear Control Special Issue on Hybrid Systems in Control*, 11(5):479–496, April 2001.

[5] J.P. Hespanha and A.S. Morse. Stability of switched systems with average dwell-time. Technical report, EE-Systems, University of Southern California, 1999.

[6] Switching in Systems and Control. *D. Liberzon*. Birkhäuser, Boston, 2003.

[7] J.E. Luntz, W. Messner, and H. Choset. Distributed manipulation using discrete actuator arrays. *Int. J. Robotics Research*, 20(7):553–583, July 2001.

[8] T. D. Murphey and J. W. Burdick. Feedback control for distributed manipulation with changing contacts. *International Journal of Robotics Research*, 23(7/8):763–782, July 2004.

[9] T. D. Murphey and J. W. Burdick. The power dissipation method and kinematic reducibility of multiple model robotic systems. *Transactions on Robotics*, Accepted for publication.

[10] K.S. Narendra, J. Balakrishnan, and M.K. Ciliz. Adaptation and learning using multiple models, switching and tuning. *IEEE Control Systems Magazine*, 15(3):37–51, 1995.

[11] H. Olsson, K.J. Astrom, C. Canudas de Wit, M. Gafvert, and P. Lischinsky. Friction models and friction compensation. *European Journal of Control*, 4(3):176–195, 1998.

[12] F. Sajjad, M. Athans, and A. Pascoal. RMMAC: A novel robust adaptive control scheme - part i: Architecture. In *Proc. 43rd IEEE Conference on Decision and Control (CDC'04)*, Paradise Island, Bahamas, December 2004.