Geometric Integration of Impact During an Orbital Docking Procedure

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***Abstract*— Simulations of orbiting bodies that experience self-impact during manuevers are known to potentially lead to numerical instability. In this paper it is demonstrated that the dynamics of an orbiting articulated body experiencing forcing and impacting can be stably simulated using variational integration. A prominent advantage of using variational integration is that conservation properties are maintained (even in the presence of external forcing) and natively can resolve impacts. Using variational integration, the configuration of the spacecraft is updated discretely to ensure that the system—subject to any applied constraints, forces, or impacts—will yield a new configuration that satisfies all conservation properties. Furthermore, variational integrators allow impacts to be easily implemented into the configuration update.**

I. Introduction

Variational integration is a numerical technique for simulating mechanical systems that preserve energy and momenta characteristics in the presence of external forcing and impacts[3], [10]. It scales well [7], [6] with the number of rigid bodies in a system, allowing one to treat interconnected bodies in an efficient manner.

In comparison to other implicit time stepping methods such as those found in [1], [2], variational integration methods represent the update map for a constrained system experiencing an impact as a root solving problem. Although this can lead—in principle—to an indeterministic number of steps in the update, it has the benefit of only returning an answer that satisfies the equations of motion, and it does so without any artificial stabilization (even in the presence of closed kinematic chains [7]). Moreover, variational integration methods provide physics-based ways of resolving multiple simultaneous impacts and yield unique solutions for systems like Newton's cradle [13] where one expects unique solutions (in contrast to [4]).

Variational integrators are in particular better for solving impacts because they involve rootsolving problems that take into account the impact condition. Typical differential algebraic techniques numerically integrate differential equations using Euler integration or the Runge-Kutta methods (or variants) and then use artificial stabilization to deal with the increase in index that occurs during impact because of the resulting closed kinematic chain. Specifically, variational integrators are a discretized form of the Euler-Lagrange equations (called the Discrete Euler-Lagrange (DEL) equations) used to solve for future configurations of a mechanical dynamical system. This update rule only relies on rootsolving, and any rootsolving technique can be used. An added advantage of using variational integration in the context of orbital docking procedures is that they preserve energy and momentum characteristics of the mechanical system which are often degraded when one uses differential algebraic techniques (because of the artificial stabilization interacting with the energetic properties of the system). For space vehical simulations, it is critical that energy and momentum are conserved to ensure the simulated orbital elements, attitude, and impacts are realistic. This work was largely motivated by interest from our colleagues at NASA in simulating robotic arms engaged in manipulation tasks while in orbit.[1]

Related work on simulating the behavior of orbiting impacting bodies is the Space Docking HIL (Hardware-In-the-Loop) Simulation [9], based on a Stewart 6-DOF (Degree-Of-Freedom) motion system. The authors derived the equation of motion for the relative movement between the "active spacecraft" (Space Shuttle) and "passive spacecraft" (ISS) in the form of a 2^{nd} order differential equation using Newton's 2^{nd} Law (as opposed to this paper's use of Euler-Lagrange equations). While their dynamic simulation software calculated the relative movement between the two spacecraft, a major problem was found: The Stewart platform's inherent phase lag resulted in unstable docking dynamics, and the controller and simulation were both contributing to the instability. The authors then had to use a phase compensation controller, adding in random gains that would cause the output frequency to increase or decrease in magnitude; hence, "correcting" for the Stewart platform's phase lag and obtaining "docking dynamics that are well replicated". Another computer simulation [17] incorporating autonomous rendezvous and docking (ARVD) capability into a six-spacecraft formation model faced similar problems: The docking-control algorithm aligned the spacecraft attitudes which, similar to [9], required gain input: During docking and undocking, [17] explained that the modeling process did "not induce unacceptable transient translational motions if the rate-feedback gains [were] set at sufficiently large values so that the trajectories during formation acquisition and rendezvous/docking phases [were] non-oscillatory". For small gain values, the authors found that dynamics describing spacecraft motion could become chaotic. This is precisely the sort of "hand tuning" that we would like to avoid, and we show in this paper that variational integration can resolve self-impact while in orbit without any tuning parameters being added to the simulation in order to obtain realistic results. As the goal of simulation is to replicate physical behavior, we believe that variational integration is a superior method because all discrete time calculations are derived directly from the laws of physics.

This paper discusses the details of simulating a simplified model of the Canadarm while docking a payload onto the International Space Station (ISS) using variational integration to simulate both

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[1] Indeed, our colleagues have found that numerical simulations of self-impact during docking have to be stabilized so much that they do not have confidence in the physical meaning of the simulation.

the free dynamics and the self-impact dynamics. This simulation involves the dynamic responses of the combined Candaram-ISS system while experiencing impacts between the payload and ISS docking port, forcing from three proportional-derivative (PD) controllers applying torque to the arms, and gravitational force from Earth. A variational integration simulation requires the system's Discrete Euler-Lagrange (DEL) equations [7], initial conditions, and a root solver (in this case, a Newton-Raphson root solver provided in Mathematica). Although we do not deal with elastic body deformations here, we showed in a previous CASE publication [12] that variational integrator methods are efficient for constrained elastic body dynamics. We later used that same methodology for a model of the human hand [5]. Elastic body assumptions will be incorporated in future work.

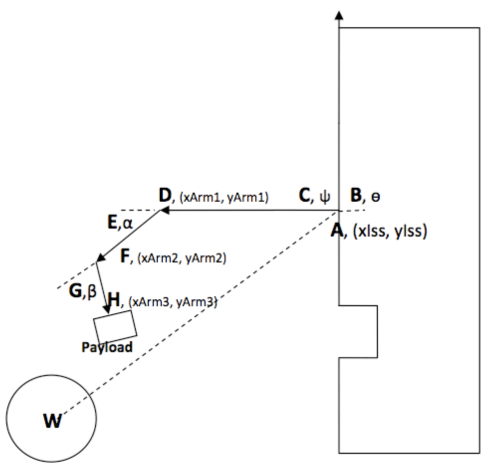


Fig. 1. Transformation Frames for a simplified model of the International Space Station and the Canadarm.

The paper is organized as follows: Section II derives the equations of motion required to set up a variational integrator in addition to the implementation of impacts within a variational integrator. Section III describes the equations and assumptions used to develop the ISS/Canadarm model. Section IV provides details on the equations used, discretization required, and rootfinding algorithm implemented in order to create the variational integrator. Section IV also covers the steps required to solve the equations of motion during impact. Section V summarizes the simulation results with plots and snapshots of the system during a docking procedure. Sections VI has conclusions and future work on elastic mechanics and optimal control for docking procedures.

II. Overview of Variational Integration

Variational integrators are formed by replacing the action integral with a discrete action sum. Let us consider a sequence of the form $(t_0, q_0), (t_1, q_1), ..., (t_n, q_n)$, where $q_k = q(t_k)$. For simplicity, consider a fixed time step, that is $h = t_{k+1} - t_k$ for all k. Now we define a *discrete* Lagrangian that approximates the action integral over one time step:

$$L_d(q_k, q_{k+1}, h) = L\left(\bar{q}, \bar{\bar{q}}\right) \approx \int_{t_k}^{t_{k+1}} L(q(\tau), \dot{q}(\tau))\, d\tau,$$

where we used the midpoint rule $\bar{q} = (q_{k+1} + q_k)/2$ and $\bar{\bar{q}} = (q_{k+1} - q_k)/h$. This leads to approximating the action integral with an action sum

$$S = \sum_{k=0}^{n-1} L_d(q_k, q_{k+1}, h). \qquad (1)$$

Minimizing (1) gives us the discrete Euler-Lagrange (DEL) equation:

$$D_2 L_d(q_{k-1}, q_k, h) + D_1 L_d(q_k, q_{k+1}, h) = 0, \qquad (2)$$

where $D_i L_d$ is the slot derivative–the derivative of L_d with respect to its i^{th} argument. This equation uses the previous two states to find the next state, thus defining a mapping of the form

$$(q_{k-1}, q_k) \rightarrow q_{k+1}.$$

In the case of external forcing—which is relevant here because of control torques being applied to the arms—the DEL equation becomes

$$\begin{aligned} &D_2 L_d(q_{k-1}, q_k, h) + D_1 L_d(q_k, q_{k+1}, h) + \\ &\quad f_d^+(q_{k-1}, t_{k-1}, q_k, t_k) + f_d^-(q_k, t_k, q_{k+1}, t_{k+1}) = 0, \end{aligned} \qquad (3)$$

where f_d^- and f_d^+ are left and right discrete forces.

Now, assume that we have determined that an impact happens during the kth time step, more precisely between t_{k-1} and t_k. This can be determined using a collision detection algorithm, a simple implementation of which would be to check for negative values of a function ϕ that describes the boundary C of the surface at each time step. Now, since we are interested mainly in what happens at the collision and not outside it, let us refer to t_{k-2} as t_o (for t_{old}), t_{k-1} as t_c (for $t_{current}$), and t_k as t_n (for t_{new}). Let the collision time be $t_1 = t_c + \alpha_1 h$, with $\alpha_1 \in [0, 1]$. We will denote the value of the configuration at the time of impact t_1 as q_1.

Applying the variational principles as before over the interval $[t_{k-1}, t_k]$, we get the following set of equations:

$$D_2 L_d(q_o, q_c, h) + D_1 L_d(q_c, q_1, \alpha_1 h) = 0, \qquad (4a)$$

$$\phi_1(q_1) = 0, \qquad (4b)$$

$$D_3 L_d(q_c, q_1, \alpha_1 h) - D_3 L_d(q_1, q_n, (1-\alpha_1)h) = 0, \qquad (4c)$$

$$\begin{aligned} &\lambda_1 \nabla \phi_1(q_1) + D_2 L_d(q_c, q_1, \alpha_1 h) \\ &\quad + D_1 L_d(q_1, q_n, (1-\alpha_1)h) = 0, \end{aligned} \qquad (4d)$$

where the unknowns are q_1, α_1, and q_n and there is no forcing (adding forcing only requires adding the same forcing terms as before to Equations 4(a,c,d)). Equation (4b) simply states that q_i must lie on the boundary at the time of impact.

III. Model of the International Space Station and Canadarm

In order to use variational integration to model the International Space Station (ISS) and the Canadarm, we need to know the inertial and forcing properties of the system. The ISS is in an elliptic, synchronous orbit with semimajor axis $a = 6,731,290$ m, mass $m_{ISS} = 344,378$ kg, and length $L_{ISS} = 73$ m. It travels at an average speed of 7706.6 m/s and has a period of 5460 s [16]. The kinetic energy of the system can be derived using equations for the body velocity [11] which include both translational and rotational energy terms. These body velocities are generally most easily calculated using homogeneous transformation matrices, represented by a 4×4 matrix G_{XY} that is the matrix representation of the rigid body transformation from frame X to frame Y. These transformations are used to translate and rotate a system of coordinate frames that are referenced to the world frame W located at Earth's center of mass (COM). We use rigid body motions to describe the relationship between every coordinate frame shown in Fig. 1. The general form of the homogeneous transformation matrix is: $G = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$ where R is a 3x3 rotation matrix and p is a 3x1 point. These transformations generalize all coordinates with respect to W. Referring to Figure 1, following the world frame is a translation from Earth's COM to frame A, centered on the International Space Station's (ISS) COM. A rotation of angle θ defines frame B, the ISS orientation. Next, a rotation of angle Ψ defines frame C, Arm1's orientation. A translation along frame C's x-axis (Arm1) to frame D, defines the location of Arm1's tip. Frame D rotates by an angle α to frame E. Next, a translation along frame E's x-axis (Arm2) to frame F, defines the tip of Arm2. The final rotation and translation are an angle β to frame G and a translation along frame G's x axis (Arm3) to frame H, the tip of Arm3. The origin of frame H is the location of the end effector which holds the payload. To transform coordinates from frame W to frame H, the transformation matrices are matrix multiplied as $G_{WH} = G_{WA}.G_{AB}.G_{BC}.G_{CD}.G_{DE}.G_{EF}.G_{FG}.G_{GH}$. Hence, the system's configuration vector is $q = [x_{ISS}, y_{ISS}, \theta, \Psi, \alpha, \beta]^T$.

The total energy of the the ISS and Canadarm can be derived as follows. The total kinetic energy is $KE = (v^b)^T.\mathcal{I}.v^b$ where v^b is the body velocity and $\mathcal{I}$ is the diagonal constant inertia tensor for each body relative to its body frame. The potential energy is simply the gravitational potential energy PE=$\frac{-Gm_e m}{r}$ where two-body dynamics are assumed, G is the gravitational constant $6.673E-11\ m^3/s^2kg$, m_e is the mass of Earth, and r is the radius between Earth and the location of the mass m for each body.

For the 2D orbital problem in this paper, only moments of inertia about the out-of-plane motion of the ISS, Canadarm sections, and payload are needed: $I_{ISS} = \frac{m_{ISS}L_{ISS}^2}{12}$, $I_{Arm1} = \frac{m_{Arm1}L_{Arm1}^2}{3}$, $I_{Arm2} = \frac{m_{Arm2}L_{Arm2}^2}{3}$, $I_{Arm3} = \frac{m_{Arm3}L_{Arm3}^2}{3}$, and $I_p = m_p L_{Arm3}^2$ where m_x is the mass of x, I_x is the inertia of x,L_x is the length of x, Arm1, Arm2, and Arm3 correspond to the three sections of the Canadarm shown in figure 1, and p denotes payload. The body velocities are: $V_{WX} = G_{WX}^{-1}.\dot{G}_{WX}$ for each body X.

Assuming the end effector has the payload and is in some position away from the docking station, three proportional-derivaitive (PD) controllers are used to guide the payload into the docking station. The PD controllers are given the configuration of the desired docking position, and then they apply torques to the three arm joints until the payload is successfully docked. Specifically, the three arm orientation angles that occur at docking position, Ψ_{docked} (2.932 rad), α_{docked} (0.741 rad), and β_{docked} (0.776 rad), are given to the PD controllers. The required torques, T_Ψ, T_α, and T_β, are then calculated using the PD control law (E.g., $T_\Psi = kp*(\Psi_{\text{desired}} - \Psi) + kd*(\dot{\Psi}_{\text{desired}} - \dot{\Psi})$) at each time step and applied to the arm joints. The proportional and derivative gain for all PD controllers were chosen to be $kp = 10$ and $kd = 2$ respectively.

The continuous force vector, f located on the right hand side of Eq. 3 represents the torques acting on the configuration vector q due to the PD controller. In vector notation, $f[T_\Psi, T_\alpha, T_\beta] = [0, 0, 0, T_\Psi, T_\alpha, T_\beta]^T$ because there are only torques applied to frames C, E, and G. Using these terms, the Lagrangian is computed as the difference between the system's total kinetic energy and potential energy.

IV. Simulation of Impacts

During docking procedures, the payload bumps into the ISS while being positioned into the docking bay by the PD controllers. In order to integrate the system's trajectory, the DEL equations in Eq. 3 and Eq. 4 are updated recursively. Given two configuration vectors, q_{k-1} and q_k Eq. 3 is root-solved until an impact is detected. When an impact is detected, Eq. 4 is used. Once q_{k+1} is known, the original away-from-impact root finder is used to compute the future configurations. Below is a summary of the steps required to detect and solve configurations before, during, and after impact. For more detail on impact equations, refer to [14].

1) Step 1: Find configuration away from impact using initial conditions: Eq. 3 is used to find all configurations before and after the three impact-configuration root finders (before and after impact).

2) Step 2: Test for Collisions: Given the configuration vector q_{k+1}; at every time step the collision detection algorithm described above is used to verify if any point of the payload has crossed an ISS boundary line ϕ (i.e. test for payload collision with the ISS).

3) Step 3: Find configuration and time at impact (q_I and t_I): After an impact is discovered, the previous two configurations of the simulation are renamed as q_{k-2} and q_{k-1} so that the next set of impact equations can refer to times occuring at t_{k-2} and t_{k-1}. The first impact-configuration root finder is used to find q_I and t_I by using the previous two times and configuration vectors: t_{k-2}, q_{k-2} and t_{k-1}, q_{k-1} respectively (where q_{k-1} is set equal to q_k, and q_{k-2} is set equal to q_{k-1}). Because the q_{k+1} from step 1 has effectively penetrated through the surface of the ISS, it is useless for further computations of the simulation and is thrown out. It is assumed that q_I is between q_{k-1} and q_k and t_I is between t_{k-1} and t_k.

4) Step 4: Find q_k and λ: Now that the impact time and configuration are known, the second impact-configuration root finder uses q_{k-1}, t_{k-1}, q_I, and t_I to find q_k (the configuration immediatly after impact). The time, t_k, corresponding to q_k is set equal to the time of the q_{k+1} configuration that penetrated through the surface. At this step of the impact procedure, the current time is set to the time t_k that the configuration was at just before impact detection.

5) Step 5: Find q_{k+1}: Equation 4 uses q_I, t_I, q_k, and t_k to solve for q_{k+1} and t_{k+1}.

6) Step 6: Return to original root finder away from impacts: Finally, the value of q_{k+1} found in step 5 is updated using Eq. 3, and the simulation away from impact continues to be computed.

V. Simulation

The following plots and animation snapshots are from a 4000-iteration (Δt=.01sec) simulation where a 100 kg payload is docked. An impact with the side of the docking bay occurs at iteration number 1250 (12.50 seconds), followed by the impact at the time of dock, or iteration number 2751 (27.51 seconds). The oscillatory behavior of each arm joint angle, shown in figures 6-9, results from the PD controllers (which, in this case, were purposefully designed to allow impacts to occur since they are known to occur during actual docking manuevers).

Figure 2 shows the orientation of the space station relative to the earth (with some scaling to make it more clear). During the docking procedure, conservation of angular momentum will dictate that the ISS rotate as the arm moves. Moreover, impacts between the arm/payload combination and the sides of the docking bay are inevitable. Figures 3 and 4 indicate the beginning and end of the simulation and the orientation of the arm relative to the space station.

During the docking procedure the system experiences several impacts due to a low gain controller being used. This reflects flexibility in the arm as well as having low torques to apply. The variational integrator has no difficulty resolving the dynamics even though there are closed kinematic chains involved in the impact. Moreover, no tuning parameters or other heuristics are needed to run the simulation.

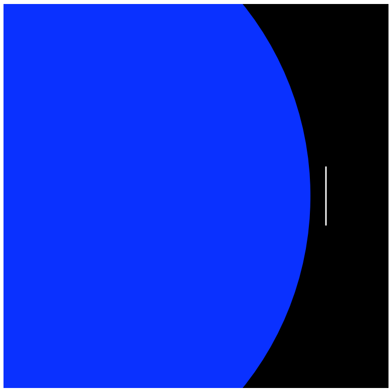


Fig. 2. ISS in Earth Synchronous Orbit

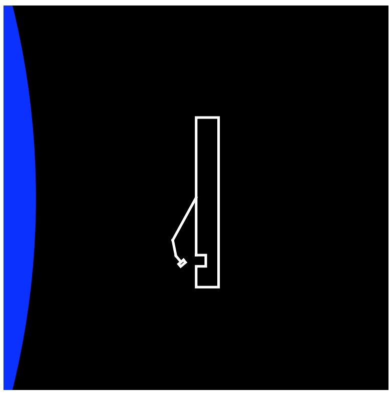


Fig. 3. Beginning of Docking Procedure: The Canadarm's end effector has grappled the 100 kg payload and begins to bring it towards the docking station

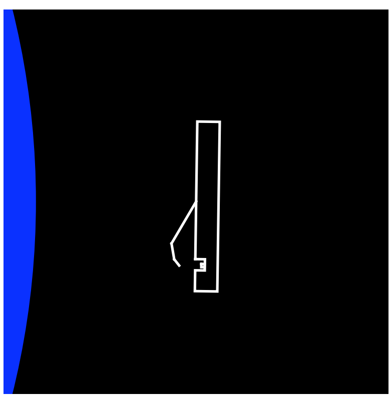
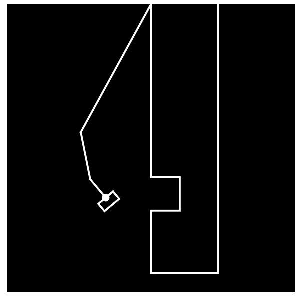
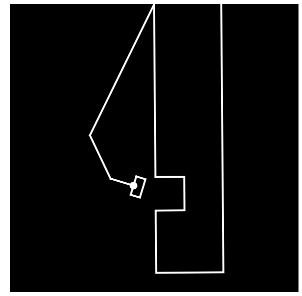


Fig. 4. End of Docking Procedure: The payload is successfully docked and the Canadarm has moved back to its original position

The rotational dynamics of the ISS and three arm segments (during the docking procedure) are described by figures 6-9. The rotational behavior of the entire system depends on torquing from arm motion in addition to the required rotational energy which keeps the ISS in an earth synchronous orbit. Figure 6 shows that the orientation of the ISS experiences small, abrupt changes when self-impacting occurs. PD control is apparent in figures 7-9 where the motion of each arm segment is shown throughout the docking procedure. As the controllers guide the payload into the docking bay the payload encounters collisions with the ISS, causing the planned trajectories of the arm sections to change; however, the PD



(a)



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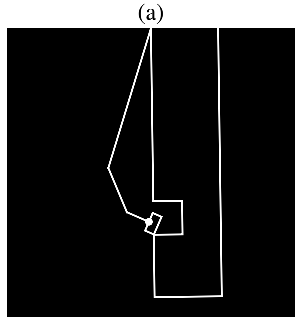

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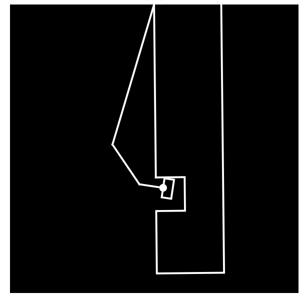

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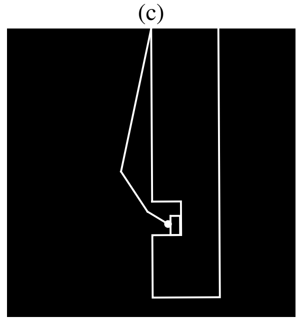

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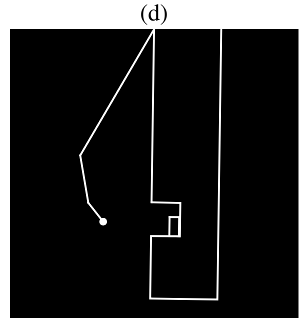

(f)

Fig. 5. Close-Up snapshots of Docking Simulation: (a) initial condition; (b) the arm starts bringing the payload in; (c) the arm experiences the first impact with the ISS; (d) the arm experiences the second impact with the ISS; (e) the docking manuever is completed; (f) the arm moves away from the payload.

controllers continue to command rotational accelerations to the arm joints that bring the payload away from the impact boundary and towards the docking bay. The non-smooth changes in the trajectories of figures 6 and 7 at the collision times of 12.5 (iteration 1250) and 27.5 (iteration 2750) seconds show that the effects of the payload impacts carry through the entire system. Although the impacts interfere with the desired PD trajectory, the control feedback is able to correct the motion until the payload is docked.

Figures 6-9 emphasize a few of the major benefits of our control; the dynamics of each of the coordinate frames which define the system may be analyzed individually, the mathematical discontinuity that occurs in the configuration trajectory at impact does not cause unrealistic behavior of the system, and during self impacts, our method yields realistic, energy and momentum-conserved dynamics.

When a standard Macbook is used to run the simulation, execution time was 3 minutes and 2 seconds while animation loading took an extra 1 minute and 2 seconds, yielding a total of 4 minutes and 4 seconds.

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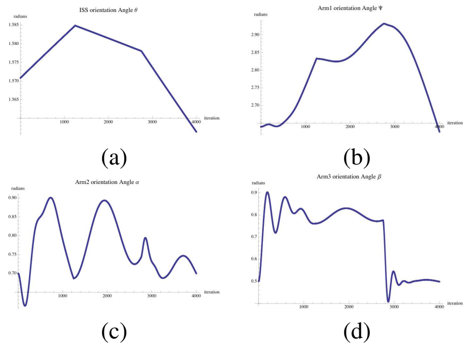

Fig. 6. Orientations during the docking procedure: (a) Orientation of the ISS; (b) Orientation of Arm1 (c) Orientation of Arm2; (d) Orientation of Arm3.

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VI. Conclusion

Robust computer models are necessary for the development of advanced space missions. The discrete algorithm used in this Candadarm-ISS docking simulation resolved the dynamic behavior of both the main body of the ISS and the arm. This simulation capability is essential both for the future of robotic spacecraft that will have moving parts, and for future missions involving spacecraft rendezvous. The use of variational integrators to derive the dynamic response of a complex system yields a plausible, energy-conserved simulation. In addition, when implementing any number of interconnected rigid bodies with closed kinematic chains, forcing controllers, or impacts into a variationally integrated simulation, the results remain realistic. Next steps include deriving optimal control laws using DMOC [15], [8]; these optimal control laws take into account the angular moment conservation as well as potential impacts and will be more efficient than the simple PD control used here. Moreover, incorporating elastic mechanics into the description of the ISS and using increasingly realistic models will be needed to verify the performance and viability of controller designs.

References

[1] N. Chakraborty, S. Berard, S. Akella, and J.C. Trinkle. An implicit time-stepping method for multibody systems with intermittent contact. In *Robotics Science and Systems (RSS)*, 2007.

[2] A. Chatterjee and A. Ruina. A new algebraic rigid-body collision law based on impulse space considerations. *Journal of Applied Mechanics*, 1998.

[3] R.C. Fetecau, J.E. Marsden, M. Ortiz, and M. West. Nonsmooth Lagrangian mechanics and variational collision integrators. *SIAM J. Applied Dynamical Systems*, 2:381–416, 2003.

[4] C. Glocker and U. Aeberhard. The geometry of Newton's cradle. *Advances in Mechanics and Mathematics*, 12:185–194, 2006.

[5] E. Johnson, K. Morris, and T. D. Murphey. *Algorithmic Foundations of Robotics VIII*, chapter A Variational Approach to Strand-Based Modeling of the Human Hand, pages 151–166. Springer-Verlag, 2010. Eds. G. Chirikjian, H. Choset, M. Morales, T. Murphey.

[6] E. Johnson and T. D. Murphey. Discrete and continuous mechanics for tree representations of mechanical systems. In *IEEE Int. Conf. on Robotics and Automation (ICRA)*, pages 1106–1111, 2008.

[7] E. Johnson and T. D. Murphey. Scalable variational integrators for constrained mechanical systems in generalized coordinates. *IEEE Transactions on Robotics*, 25(6):1249–1261, 2009.

[8] O. Junge, J. E. Marsden, and S. Ober-Blobaum. Discrete mechanics and optimal control. In *IFAC Congress*, 2005.

[9] Han Junwei, Huang Qitao, and Chang Tongli. Research on space docking hil simulation system based on stewart 6-dof motion system. In *7th JFPS International Symposiumon Fluid Power*, 2008.

[10] C. Kane, E.A. Repetto, M. Ortiz, and J.E. Marsden. Finite element analysis of nonsmooth contact. *Comput. Methods Appl. Mech. Engrg.*, 180:1–26, 1999.

[11] R.M. Murray, Z. Li, and S.S. Sastry. *A Mathematical Introduction to Robotic Manipulation*. CRC Press, 1994.

[12] K. Nichols and T. D. Murphey. Variational integrators for constrained cables. In *IEEE Int. Conf. on Automation Science and Engineering (CASE)*, pages 802–807, 2008.

[13] V. Seghete and T. D. Murphey. Multiple instantaneous collisions in a variational framework. In *IEEE Int. Conf. on Decision and Control (CDC)*, pages 5015 – 5020, 2009.

[14] V. Seghete and T. D. Murphey. Variational solutions to simultaneous collisions between multiple rigid bodies. In *IEEE Int. Conf. on Robotics and Automation (ICRA)*, 2010.

[15] K. Snyder and T. D. Murphey. Second-order DMOC using projections. In *IEEE Int. Conf. on Decision and Control (CDC)*, Submitted.

[16] Wikepedia the free encyclopedia. International space station. 2010.

[17] P.K.C. Wang and F.Y. Hadaegh. Formation ying of multiple spacecraft with autonomous rendezvous and docking capability. In *IET Control Theory Appl.*, page 494 504, 2007.