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Topology-Based Variational Integration of Degenerate Interconnected Mechanical Systems

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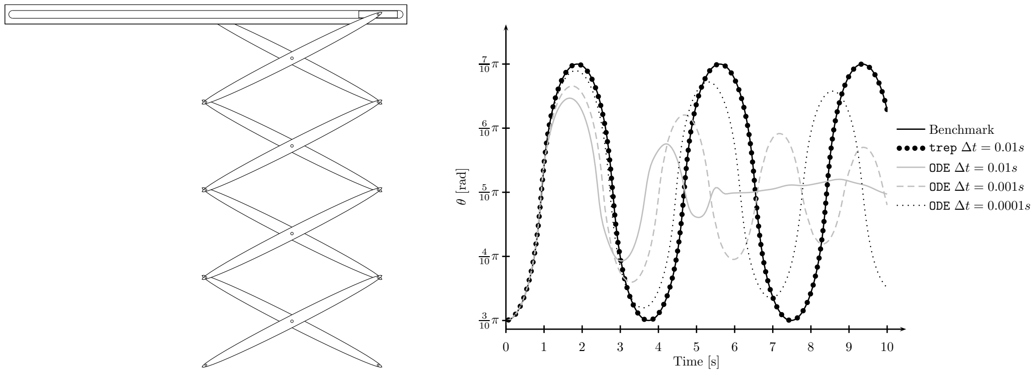
Variational integration methods, like those found in [1, 2], use the stationary action principle as a foundation for numerical integration that does not involve differential equations. This approach has several advantages–known conservation properties (such as guarantees about conservation of momenta and the Hamiltonian) as well as guaranteed convergence to the correct trajectory. More importantly, variational integration techniques exactly simulate a *modified Lagrangian* system where the Lagrangian is a perturbation of the original Lagrangian. For low-dimensional systems, the advantages of variational integrators are often comparatively academic in nature–it can simply be a matter of personal preference to numerically treat a system in a variational setting instead of a more traditional differential algebraic setting. However, as systems become more complex other advantages become evident.



(a) (b)

Figure 1: Using the mechanical topology of a system to organize calculations leads to scalable variational integration in generalized coordinates just as it does in the continuous setting. The scissor lift (a) is a constrained system with only one degree-of-freedom but can be simulated as a $6n$ DOF system with $6n-1$ constraints. Comparisons with the Open Dynamics Engine (ODE) in (c) show that the variational integration preserves the structure of the mechanics even with all the interconnections.

Consider the “scissor lift” system shown in Fig. 1(a). It has 48 degrees of freedom (DOF) along with 47 constraints. Hence, although one can simulate it as a single DOF system for benchmarking purposes, one can use it to test constrained simulation techniques. Figure 1(b) shows the single DOF versus time using the benchmark (generated in *Mathematica* by integrating the 1 DOF representation of the scissor-lift), Open Dynamics Engine (*http://www.ode.org*), and topology-based variational integrators [3]. The solid black line represents the benchmark while the dotted line represents the high dimensional variational integrator. Additionally, three simulations from the Open Dynamics Engine are shown using time steps of 10^{-3}, 10^{-4}, and 10^{-5} seconds. At these time steps the Open Dynamics Engine diverges quickly; more importantly, smaller time steps are *not possible* because one runs into machine precision calculation problems. Hence, Open Dynamics Engine cannot be expected to simulate this system well.[1] The

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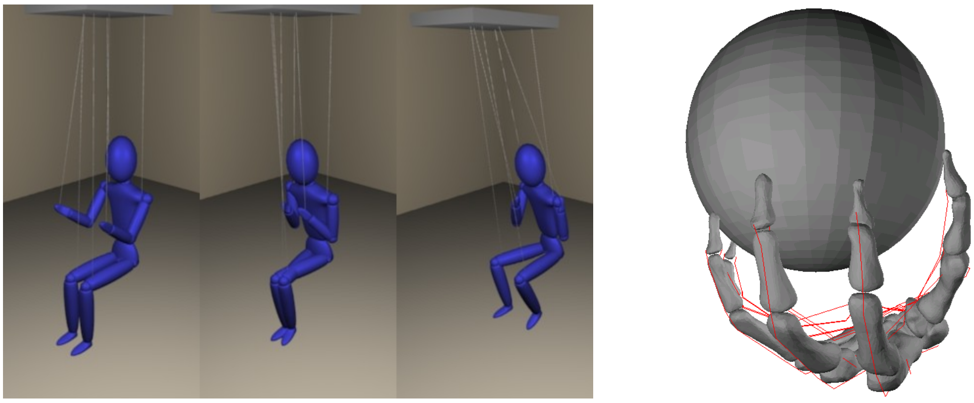
[1] It is important to note that Open Dynamics Engine is only being used for illustration; it was not *designed* to simulate interconnected systems and is very clear about this in its documentation. It is designed for ease of use in animation and other situations that do not rely on physical fidelity.



(a) (b)

Figure 2: Degenerate mechanical systems have components that are inertialess or nearly inertialess. In (a), a marionette has strings that can be modeled without mass and in (b) a tendon-articulated skeletal hand holding an object can be modeled treating the tendons as massless cables. In both cases the resulting Lagrangian system is degenerate and is characterized by multiple dynamically coupled, degenerate closed kinematic chains. Nevertheless, variational integration techniques are fast and stable for these systems (e.g., leading to real-time simulation for the marionettes).

topology-based variational integration techniques developed in [3] allow efficient computation of the underlying discrete Euler-Lagrange equations. Moreover, variational integration methods are particularly useful in evaluating *degenerate* Lagrangian systems. One can show, using a discrete Dirac Structure formulation [2], that degeneracies like those seen in marionette simulation [4] in Fig. 2 and in hand simulation [5] in Fig. 2(b) can be stably numerically integrated using variational integration methods. More importantly, these methods are stable without ad-hoc introduction of artificial stabilization (which confounds system identification, among other things). Lastly, simultaneous collisions can also be resolved in a variational context [6], an important feature of grasping problems where the fingers are capable of simultaneously coming in and out of contact with an object. Moreover, in a variational setting one can improve upon prior results in simultaneous impact problems; for instance, Newton's cradle has unique solutions in a variational context [6].

Depending on the system of interest, variational integration techniques provide more than philosophical advantages over their differential algebraic counterparts; in the examples discussed here they provide numerically stable techniques that are simple to implement and are often real-time amenable. Future work includes embedded system design for complex dynamical systems like the hand in Fig. 2(b) based on discrete variational principles.

References

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